Generic Approach to Determine Optimum Aeroelastic Characteristics for Composite Forward-Swept-Wing Aircraft

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Aeroelastic tailoring, a concept which is critical to the development of forward-swept-wing aircraft, is presented as a multivariable optimization problem in which all of the variables have to be considered—a departure from the current practice in which the fiber orientation angle seems to be the only variable used in the tailoring process. A transformation of the aeroelastic equations of motion for a composite swept wing reveals that the critical aeroelastic characteristics for flutter and divergence are expressible in terms of three bounded generic stiffness variables and the fiber orientation angle. A variation of these variables within their various limits permits a view of the complete continuum of the critical aeroelastic parameters for all composite materials. The results for aeroelastic divergence presented in this paper show that 1) divergence can be eliminated for a) any forward-swept angle and b) a forward-swept wing whose fiber orientation angles are swept back relative to the spanwise reference axis; and 2) an optimum aeroelastically tailored forward-swept wing is one that uses different composite materials oriented at various angles (a configuration that also enhances the wing's strength).

Nomenclature

a_0	= lift curve slope
c	= wing chord
D^*	= lamina's generalized rigidity ratio
e	= wing section aerodynamic center offset (+aft)
h	= elastic axis upward displacement
ℓ	= wing semispan
q	= dynamic pressure
r	= lamina's principal rigidity ratio
t_0	= time
(x,y)	= swept wing's reference axes with the origin at the root
(x_θ,y_θ)	= lamina principal axes
α	= pitching displacement
ϵ	= lamina' generalized Poisson's ratio
θ	= reference fiber orientation angle relative to wing's rearward normal
Λ	= spanwise reference axis sweep angle (+for sweepback)
λ	= taper ratio
τ	= wing skin thickness

Subscripts

a	= reference lamina quantities
cr	= critical quantities
D	= divergence quantities
i	= lamina quantities
r	= wing root quantities
$\bar{( )}$	= quantities in the affine space

Introduction

FOR over four decades attempts to capitalize on the superior aerodynamic and configurational characteristics of forward-swept-wing aircraft have been consistently frustrated by an aeroelastic instability known as divergence. In the middle of the last decade, efforts to understand and remedy this phenomenon began to yield results when studies by Krone[1] revealed (using a numerical approach) that a composite forward-swept wing can be aeroelastically tailored to overcome the divergence problem.

This conclusion inspired the studies by Weisshaar,[2,3] in which he proposed an analytical model for the mechanism of aeroelastic tailoring involved. Although he did not use state-of-the-art aerodynamics, wind tunnel tests conducted by Sherrer et al.[4] seem to verify the model. A considerable amount of work has also been done by other investigators[5-10] (most of them using numerical methods) to develop this concept. As a result, numerous computer programs now exist for optimizing the tailoring process.

All of these studies, however, were done on aeroelastic models having so many variables (aerodynamic and elastic) whose relative importance and bounds are unknown (except the fiber orientation angle). This probably explains why the fiber orientation angle is practically the only variable considered in aeroelastic tailoring. This state of affairs prevents any convenient comparisons between various candidate composite materials in a particular situation. In fact, all of the conclusions presented by the studies in aeroelastic tailoring thus far are based on the results from a few materials and may not necessarily represent the general trend for all composite materials. Therefore, a need exists for a unified analysis in which the critical aeroelastic variables can be presented for the entire spectrum of composite materials. This would permit an aeroelastic tailoring that considers all of the variables.

In this paper, a transformation of the aeroelastic equations of motion for a composite forward-swept wing, using a model similar to Weisshaar's,[2] reveals that the critical aeroelastic characteristics can be expressed in terms of three bounded, generic stiffness variables and the fiber orientation angle. A variation of these variables within their various limits permits plots of the continuum of divergence characteristics for all composite materials. This continuum exposes certain important facts that previous studies were unable to show. They include the following:

1) Aeroelastic divergence can be eliminated for a) any forward-swept angle, and b) a forward-swept wing whose fibers are oriented at angles swept-back from the spanwise reference axis.

2) An optimum aeroelastically tailored forward-swept wing is one that uses different composite materials oriented at

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various angles (a fact that should enhance the wing's strength).

While the analysis for the bending-torsion flutter by the author is in progress, similar ideas have been used in Refs. 11-14 for panel static instability and vibration, and in Refs. 15-17 for panel flutter.

Problem Statement

This analysis considers a high aspect ratio forward-swept wing, idealized by a box beam whose planform is shown in Fig. 1a, deriving all of its bending and torsional stiffness from the laminated composite upper and lower skins,† and subjected to an airflow V. The fiber orientation relative to the spanwise reference axis is shown in Fig. 1b.

Aeroelastic Equations of Motion

Utilizing the same displacement assumptions as those used by Weisshaar[2] and Housner and Stein,[5] the constant chord swept wing's aeroelastic equations of motion derived using the Lagrangian approach are, in Weisshaar's notation,

$$B_{33}\alpha'' - A_{22}v_0'' + B_{22}h''' = 0$$

$$S\alpha^{iv} - GJ_0\alpha'' - B_{33}v_0'' + K_0h''' = t + I_\alpha\ddot{\alpha} - mx_\alpha\ddot{h}$$

$$EI_0h^{iv} - K_0\alpha''' - B_{22}v_0''' = p + m(\ddot{h} - \ddot{\alpha}x_\alpha) \tag{1}$$

The boundary conditions are, for $y=0$,

$$v_0 = h = h' = \alpha = \alpha' = 0 \tag{2}$$

and for $y=\ell$,

$$-S\alpha''' + GJ_0\alpha' + B_{33}v_0' - K_0h'' = 0$$

$$B_{33}\alpha' + A_{22}v_0' - B_{22}h'' = 0$$

$$EI_0h'' - K_0\alpha' - B_{22}v_0' = 0$$

$$EI_0h''' - K_0\alpha'' - B_{22}v_0'' = \alpha' = 0 \tag{3}$$

where

$$(\ )' = \frac{\partial}{\partial y}(\ ),\ (\dot{\ }) = \frac{\partial}{\partial t_0}(\ )$$

v_0 is the initial y-axis displacement of the wing; m the mass per unit length, x_α the center of mass offset from reference axis (+aft), and I_α the section mass moment of inertia per unit length about the reference axis of the wing,

$$EI_0 = c\sum_{i=1}^{N}\bar{Q}_{22}^{(i)}\beta_i, \qquad GJ_0 = 4c\sum_{i=1}^{N}\bar{Q}_{66}^{(i)}\beta_i$$

$$S = \frac{c^3}{12}\sum_{i=1}^{N}\bar{Q}_{22}^{(i)}\beta_i, \qquad K_0 = 2c\sum_{i=1}^{N}\bar{Q}_{26}^{(i)}\beta_i$$

$$A_{22} = c\sum_{i=1}^{N}\bar{Q}_{22}^{(i)}\tau_i, \qquad B_{22} = c\sum_{i=1}^{N}\bar{Q}_{22}^{(i)}\delta_i$$

$$B_{33} = 2c\sum_{i=1}^{N}\bar{Q}_{26}^{(i)}\delta_i \tag{4}$$

where $\bar{Q}_{ij}$, the laminate elastic constants,[18] are functions of the specially orthotropic elastic constants, Q_{ij}, and the fiber orientation angle, θ, and N is the number of composite material layers.

†Stiffness from spars, ribs, webs, etc., could be added algebraically to those derived here.

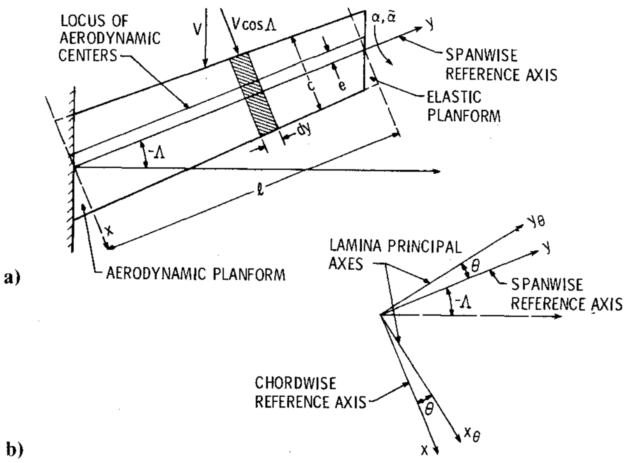

Fig. 1 Slender swept-wing planform and lamina orientation.

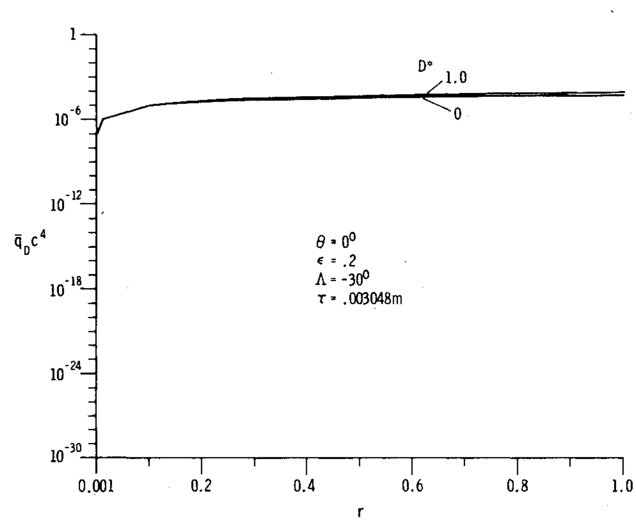

Fig. 2 Generic divergence dynamic pressure vs r ($\theta=0$ deg).

$$\beta_i \triangleq \int_{z_i}^{z_{i+1}} z^2\,dz, \qquad \delta_i \triangleq \int_{z_i}^{z_{i+1}} z\,dz \tag{5}$$

where τ_i is the lamina thickness; z_i, z_{i+1} are its lower and upper surface coordinates, respectively; p is the aerodynamic force per unit length (+upward); and t the torque per unit length (+nose up). Reference 2 derived a version of Eqs. (1-3) in which time dependence and S terms were omitted.

Transformations and Generic Variables

Consider the affine transformations

$$(h, v_0, \alpha) \equiv (Q_{11}Q_{22})_a^{-1/2}(\bar{h}, \bar{v}_0, \bar{\alpha}) \tag{6}$$

where $(Q_{11}Q_{22})_a$ is the product of the principal axes elastic constants of a reference lamina.

Equation (6) transforms Eqs. (1-3) into the following:

$$\bar{B}_{33}\bar{\alpha}'' - \bar{A}_{22}\bar{v}_0'' + \bar{B}_{22}\bar{h}''' = 0$$

$$\bar{S}\bar{\alpha}^{iv} - \overline{GJ}_0\bar{\alpha}'' - \bar{B}_{33}\bar{v}_0'' + \bar{K}_0\bar{h}''' = \bar{t} + \bar{I}_\alpha\ddot{\bar{\alpha}} - \bar{m}x_\alpha\ddot{\bar{h}}$$

$$\overline{EI}_0\bar{h}^{iv} - \bar{K}_0\bar{\alpha}''' - \bar{B}_{22}\bar{v}_0''' = \bar{p} + \bar{m}(\ddot{\bar{h}} - \ddot{\bar{\alpha}}x_\alpha) \tag{7}$$

The corresponding boundary conditions are, for $y=0$,

$$\bar{v}_0 = \bar{h} = \bar{h}' = \bar{\alpha} = \bar{\alpha}' = 0 \tag{8}$$

and for $y=\ell$,

$$-\bar{S}\bar{\alpha}''' + \overline{GJ}_0\bar{\alpha}' + \bar{B}_{33}\bar{v}_0' - \bar{K}_0\bar{h}'' = 0$$
$$\bar{B}_{33}\bar{\alpha}' + \bar{A}_{22}\bar{v}_0' - \bar{B}_{22}\bar{h}'' = 0$$
$$\overline{EI}_0 h'' - \bar{K}_0\bar{\alpha}' - \bar{B}_{22}\bar{v}_0' = 0$$
$$\overline{EI}_0\bar{h}''' - \bar{K}_0\bar{\alpha}'' - \bar{B}_{22}\bar{v}_0'' = \bar{\alpha}' = 0 \tag{9}$$

where

$$\overline{EI}_0 = \frac{c}{8}\sum_{i=1}^{N}\Big[(3-4\cos 2\theta_i + \cos 4\theta_i)\frac{1}{r_i}$$
$$+ (3+4\cos 2\theta_i + \cos 4\theta_i) r_i + 2(1-\cos 4\theta_i) D_i^*\Big]\bar{\beta}_i$$
$$\overline{GJ}_0 = \frac{c}{2}\sum_{i=1}^{N}\Big\{\Big(\frac{1}{r_i}+r_i\Big)(1-\cos 4\theta_i)$$
$$+2D_i^*[(1-2\epsilon_i)+\cos 4\theta_i]\Big\}\bar{\beta}_i$$
$$\bar{K}_0 = \frac{c}{4}\sum_{i=1}^{N}\Big[\frac{1}{r_i}(2\sin 2\theta_i - \sin 4\theta_i)$$
$$-r_i(2\sin 2\theta_i + \sin 4\theta_i) + 2D_i^*\sin 4\theta_i\Big]\bar{\beta}_i$$
$$\bar{S} = \frac{c^2}{12}\overline{EI}_0, \quad \bar{A}_{22} = \Big(\frac{\bar{\tau}_i}{\bar{\beta}_i}\Big)\overline{EI}_0$$
$$\bar{B}_{22} = \Big(\frac{\bar{\delta}_i}{\bar{\beta}_i}\Big)\overline{EI}_0, \quad \bar{z}B_{33} = \Big(\frac{\bar{\delta}_i}{\bar{\beta}_i}\Big)\bar{K}_0 \tag{10}$$
$$\bar{t} = t(\bar{\alpha},\bar{h},\bar{v}_0), \quad \bar{p} = p(\bar{\alpha},\bar{h},\bar{v}_0)$$
$$(\bar{m},\bar{I}_\alpha) \equiv (Q_{11}Q_{22})_a^{-1/2}(m,I_\alpha)$$
$$(\bar{\beta}_i,\bar{\delta}_i,\bar{\tau}_i) \equiv \Big[\frac{(Q_{11}Q_{22})_i}{(Q_{11}Q_{22})_a}\Big]^{1/2}(\beta_i,\delta_i,\tau_i) \tag{11}$$

and

$$r_i = \Big(\frac{Q_{22}}{Q_{11}}\Big)_i^{1/2}, \quad D_i^* = \frac{(Q_{12}+2Q_{66})_i}{(Q_{11}Q_{22})_i^{1/2}}$$
$$\epsilon_i = (Q_{12})_i / [D_i^*(Q_{11}Q_{22})_i^{1/2}] \tag{12}$$

Equations (7) are the aeroelastic equations of motion, Eqs. (8) and (9), the associated boundary conditions, and Eqs. (10-12) define the generic variables in the affine space.

The key to this analysis are the generic variables r_i, D_i^*, and ϵ_i, called the principal rigidity ratio (PRR), the generalized rigidity ratio (GRR), and the generalized Poisson's ratio (GPR), respectively. Their values for different composite materials are shown in Table 1. Although they have been studied and used in Refs. 11-17, their derivation in this work is effected using a different set of transformations. Their limits for all composite materials have been established as follows: $0<r_i\leq 1$, $0\leq D_i^*\leq 1$, and $0.12\leq\epsilon_i\leq 0.65$ (similar to the range of Poisson's ratio for isotropic materials). This means that on solving Eqs. (7) subject to Eqs. (8) and (9), the critical aeroelastic characteristics (including divergence and flutter) for the swept wing can be expressed in terms of these bounded variables and the fiber orientation angle. Thus, aeroelastic tailoring is reduced to selecting the optimum points of five-dimensional space curves.

When the variable $\bar{v}_0$ is eliminated using the axial loading arguments,[2] Eqs. (7-9) reduce to the following:

$$\bar{S}\bar{\alpha}^{iv} - \overline{GJ}\bar{\alpha}'' + \bar{K}\bar{h}''' = \bar{t} + \bar{I}_\alpha\ddot{\bar{\alpha}} - \bar{m}x_\alpha\ddot{\bar{h}}$$
$$\overline{EI}\bar{h}^{iv} - \bar{K}\bar{\alpha}''' = \bar{p} + \bar{m}(\ddot{\bar{h}} - \ddot{\bar{\alpha}}x_\alpha) \tag{13}$$

Boundary conditions are, for $y=0$,

$$\bar{h} = \bar{h}' = \bar{\alpha} = \bar{\alpha}' = 0 \tag{14}$$

and for $y=\ell$,

$$-\bar{S}\bar{\alpha}''' + \overline{GJ}\bar{\alpha}' - \bar{K}\bar{h}'' = 0$$
$$\overline{EI}\bar{h}'' - \bar{K}\bar{\alpha}' = 0, \qquad \overline{EI}\bar{h}''' - \bar{K}\bar{\alpha}'' = \alpha' = 0 \tag{15}$$

where

$$\overline{EI} = \overline{EI}_0 - \frac{\bar{B}_{22}^2}{\bar{A}_{22}}, \quad \overline{GJ} = \overline{GJ}_0 - \frac{\bar{B}_{33}^2}{\bar{A}_{22}}, \quad \bar{K} = \bar{K}_0 - \frac{\bar{B}_{22}\bar{B}_{33}}{\bar{A}_{22}} \tag{16}$$

Aerodynamics

The aerodynamics required to solve these aeroelastic equations depends on whether the problem is subsonic, transonic, or supersonic, divergence or flutter, and generally varies in levels of sophistication.[19]

Due to the critical nature of the divergence problem for forward-swept-wing aircraft, the remainder of this analysis is devoted to this instability. Furthermore, simple aerodynamics shall be employed in order to establish trends. Since the generic perturbation aerodynamic forces and moments responsible for divergence are caused by the perturbed generic displacements $\bar{h}$ and $\bar{\alpha}$, the expressions for $\bar{p}$ and $\bar{t}$, using the aerodynamic strip theory and Fig. 1a, are:

$$\bar{p}(y) = \bar{q}ca_0\cos^2\Lambda(\bar{\alpha} - \bar{h}'\tan\Lambda)$$
$$\bar{t}(y) = \bar{q}cea_0\cos^2\Lambda(\bar{\alpha} - \bar{h}'\tan\Lambda) \tag{17}$$

where $\bar{q}$, the generic dynamic pressure, is defined by

$$\bar{q} = (\tfrac{1}{2}\rho V^2)/(Q_{11}Q_{22})_a^{1/2} \tag{18}$$

and a_0, the lift curve slope, is given by

$$a_0 = 2\pi[\mathcal{R}/(\mathcal{R}+4\cos\Lambda)] \tag{19}$$

ρ, Λ, e, and $\mathcal{R}$ are the air mass density, the wing's sweep angle, the distance between the wing's spanwise reference axis and the line of the aerodynamic centers, and the wing's aspect ratio, respectively.

Table 1 Generic parameters (stiffness data taken from Ref. 22)

Variable	r	D^*	ϵ
Range for all materials	$0<r\leq 1.0$	$0\leq D^*\leq 1.0$	$0.14\leq\epsilon\leq 0.65$
Graphite/epoxy	0.324	0.329	0.21
Graphite/epoxy (AS/3501)	0.258	0.299	0.26
GY 70/epoxy	0.488	0.238	0.62
E-glass/epoxy	0.052	0.050	0.28
Boron/epoxy	0.262	0.254	0.22
Graphite/aluminum	0.447	0.915	0.15

Divergence Analysis

For divergence, Eqs. (13) reduce to

$$\overline{EI}\bar{h}^{iv} - \bar{K}\alpha''' = \bar{q}ca_0\cos^2\Lambda(\bar{\alpha}-\bar{h}'\tan\Lambda)$$

$$\bar{S}\alpha^{iv} - \overline{GJ}\bar{\alpha}'' + \bar{K}\bar{h}''' = \bar{q}cea_0\cos^2\Lambda(\bar{\alpha}-\bar{h}'\tan\Lambda) \qquad (20)$$

Although it is the author's belief that Eqs. (20) should be solved subject to the boundary conditions in Eqs. (15), the terms containing $\bar{S}$ are neglected here to make the analysis a little easier (the effects of $\bar{S}$ on the results of this analysis shall be examined in future studies). Hence Eqs. (20) reduce to

$$\overline{EI}\bar{h}^{iv} - \bar{K}\bar{\alpha}''' = \bar{q}ca_0\cos^2\Lambda(\bar{\alpha}-\bar{h}'\tan\Lambda)$$

$$-\overline{GJ}\bar{\alpha}''' + \bar{K}\bar{h}''' = \bar{q}cea_0\cos^2\Lambda(\bar{\alpha}-\bar{h}'\tan\Lambda) \qquad (21)$$

subject to the following boundary conditions (a result of dropping "S" from the energy expression).

For $y=0$,

$$\bar{h} = \bar{h}' = \bar{\alpha} = 0 \qquad (22)$$

For $y=\ell$,

$$\overline{EI}\bar{h}'' - \bar{K}\alpha' = \overline{EI}\bar{h}''' - \bar{K}\bar{\alpha}'' = \overline{GJ}\bar{\alpha}' - \bar{K}\bar{h}'' = 0 \qquad (23)$$

Using methods similar to those outlined in Refs. 2, 20, and 21, Eqs. (21) subject to Eqs. (22) and (23) (modified to include the wing's linear taper effects), are solved to obtain the following expressions for the critical aeroelastic characteristics for swept wings

$$\bar{q}_D = \frac{K_1(1-k_r g_r)}{a_0 c_r \ell^3 \cos^2\Lambda} \times \left\{ \overline{EI}_r \Big/ \left[\frac{1-k_r\tan\Lambda}{(\ell/e_r)(\overline{GJ}_r/\overline{EI}_r)} - K_2(\tan\Lambda - g_r) \right] \right\} \qquad (24)$$

$$\tan\Lambda_{cr} = \frac{g_r + (1/K_2)(e_r/\ell)(\overline{EI}_r/\overline{GJ}_r)}{1+(g_r/K_2)(e_r/\ell)} \qquad (25)$$

where $\bar{q}_D$ is the generic divergence dynamic pressure and Λ_{cr} is the minimum (maximum negative) sweep angle at which divergence ceases to exist. K_1 and K_2, tabulated in Ref. 2, are functions of the wing's taper ratio, λ. For $\lambda=0.2$, $K_1=2.83$, $K_2=0.614$; for $\lambda=0.5$, $K_1=2.73$,

$$K_2=0.497;\ \text{for}\ \lambda=1.0,\ K_1=2.47,\ K_2=0.390 \qquad (26)$$

$$k_r = \bar{K}_r/\overline{EI}_r, \quad g_r = \bar{K}_r/\overline{GJ}_r \qquad (27)$$

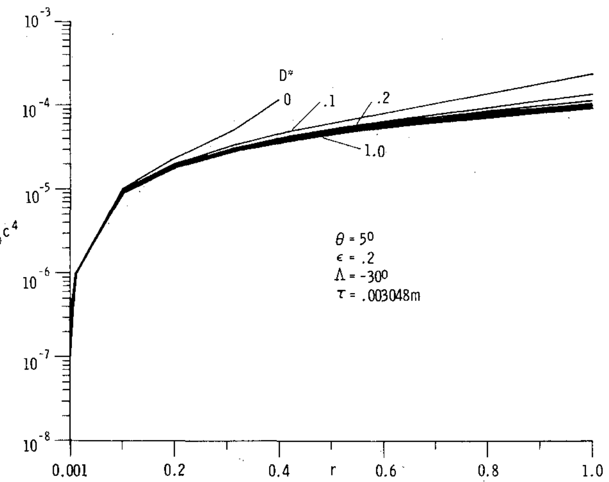

Fig. 3 Generic divergence dynamic pressure vs r ($\theta = 5$ deg).

Divergence Continuum

For illustrative purposes, the analysis considers a constant chord wing with the following properties: $AR=6$, $c/e=10$, ratio of box-beam depth to skin thickness = 20:1, skin thickness (all tailorable, i.e., $N=1$) = 0.003048 m. Figures 2-8 depict the continuum of the critical generic divergence dynamic pressure of a 30-deg forward-swept wing for all composite materials. It is seen that $\bar{q}_D$, where it exists, increases with increasing r for $\theta<92$ or >140 deg, decreases

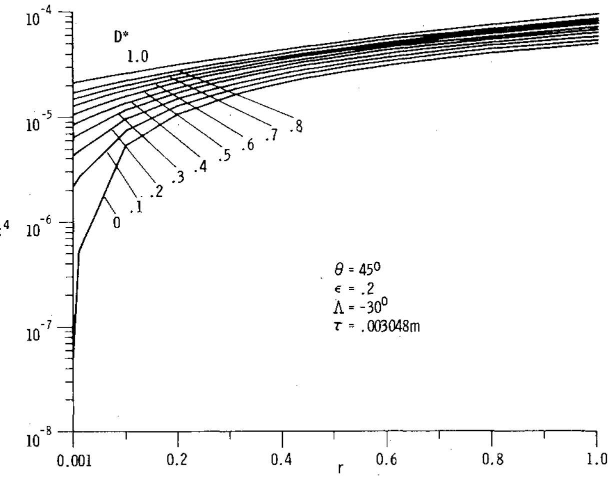

Fig. 4 Generic divergence dynamic pressure vs r ($\theta = 45$ deg).

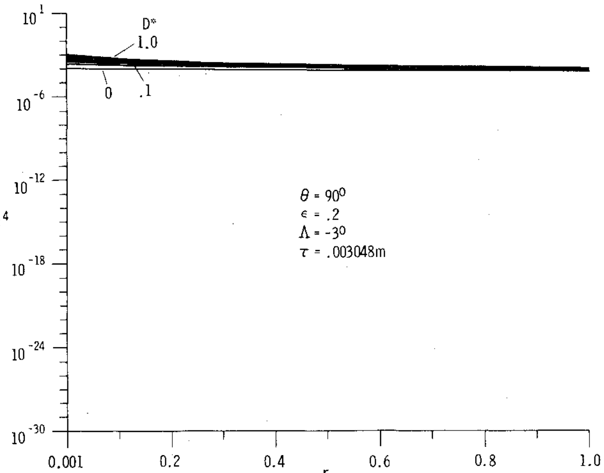

Fig. 5 Generic divergence dynamic pressure vs r ($\theta = 90$ deg).

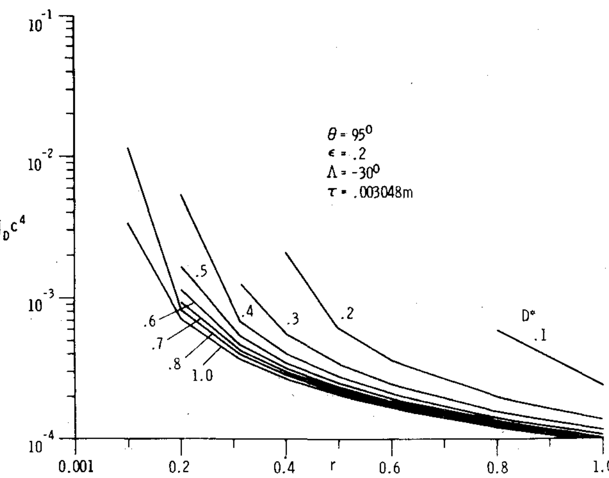

Fig. 6 Generic divergence dynamic pressure vs r ($\theta = 95$ deg).

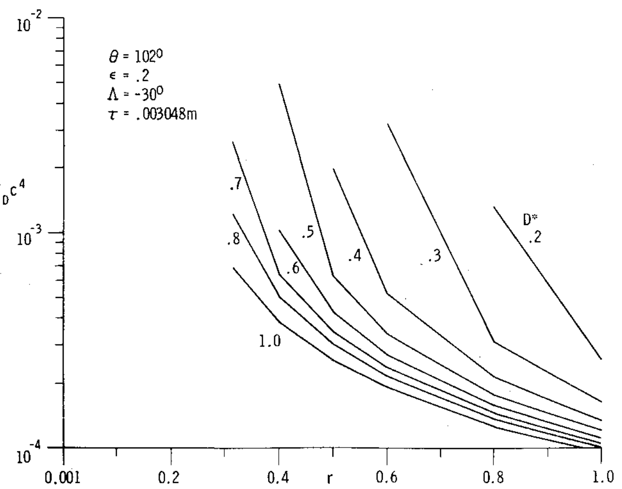

Fig. 7 Generic divergence dynamic pressure vs r ($\theta = 102$ deg).

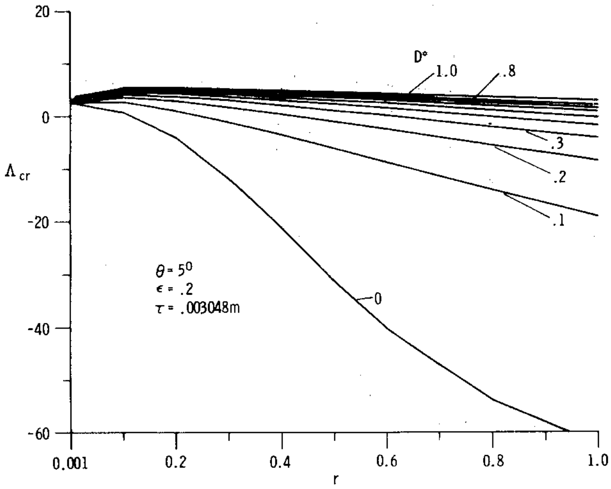

Fig. 10 Divergence-free wing sweep angle vs r ($\theta = 5$ deg).

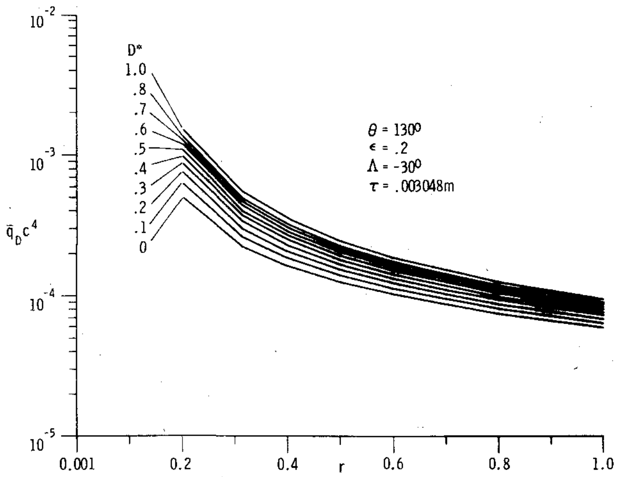

Fig. 8 Generic divergence dynamic pressure vs r ($\theta = 130$ deg).

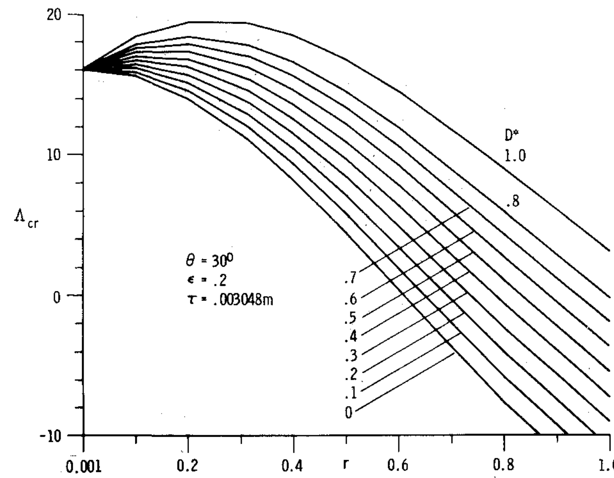

Fig. 11 Divergence-free wing sweep angle vs r ($\theta = 30$ deg).

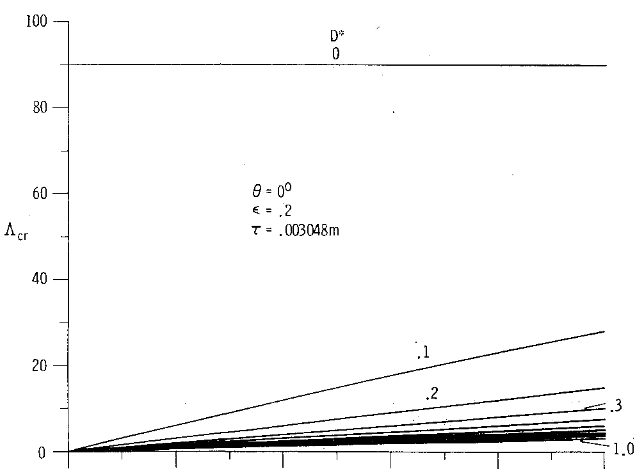

Fig. 9 Divergence-free wing sweep angle vs r ($\theta = 0$ deg).

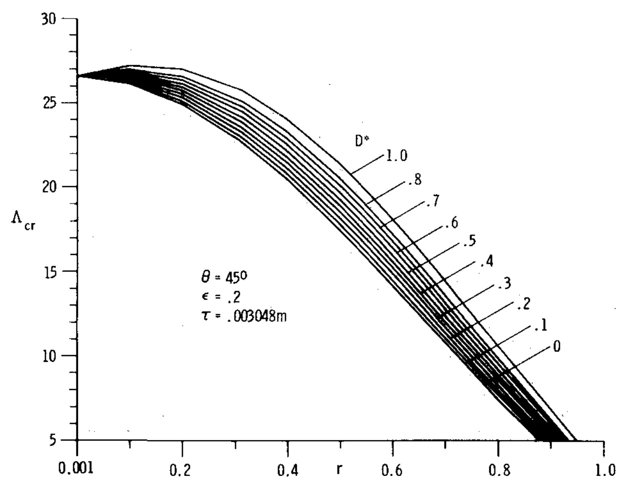

Fig. 12 Divergence-free wing sweep angle vs r ($\theta = 45$ deg).

with increasing r for 92 deg $<\theta<$ 140 deg, increases with increasing D^* for $r<0.1$ ($\theta<30$ deg), 30 deg $<\theta<$ 92 deg, 130 deg $<\theta<$ 180 deg, and 0 deg (180 deg) and decreases with increasing D^* for $r>0.1$ ($\theta<30$ deg) and 92 deg $<\theta<$ 130 deg. Figures 9-19 display the continua of the minimum swept angle for which divergence ceases to exist. From these figures it is seen that Λ_{cr} increases with increasing r for 92 deg $<\theta<$ 180 deg ($r<0.2$), $r<0.1$ (0 deg $<\theta<$ 50 deg, $D^*>0.1$), $r\leq 1$ ($\theta<8$ deg, $D^*>0.6$) and 0 deg (180 deg), decreases with increasing r for 8 deg $<\theta<$ 92 deg ($r>0.2$), increases with increasing D^* for 0 deg $<\theta<$ 50 deg and 92 deg $<\theta<$ 130 deg, decreases with increasing D^* for 50 deg $<\theta<$ 92 deg, 130 deg $<\theta<$ 180 deg and 0 deg (180 deg), and varies very little with ϵ[i.e., $\bar{q}_D$ and Λ_{cr} are approximately functions of only three variables (r, D^*, θ)].

While Fig. 9 predicts that a wing with zero fiber orientation has to be swept back to avoid divergence, Fig. 10 indicates that for swept-back fibers ($\theta<90$ deg), as $D^*\rightarrow 0$ and $r\rightarrow 1$, a

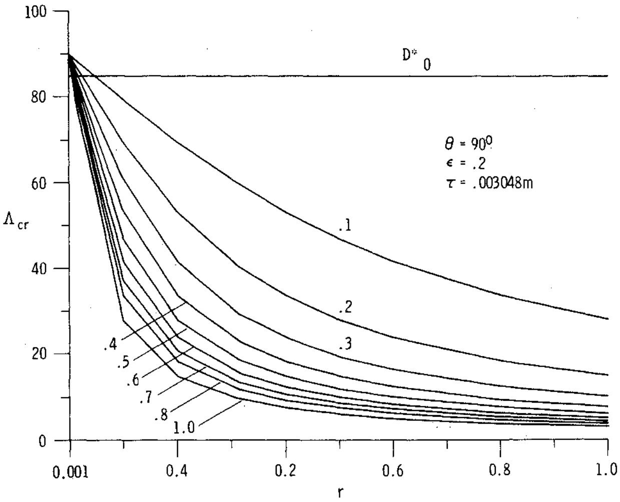

Fig. 13 Divergence-free wing sweep angle vs r ($\theta = 90$ deg).

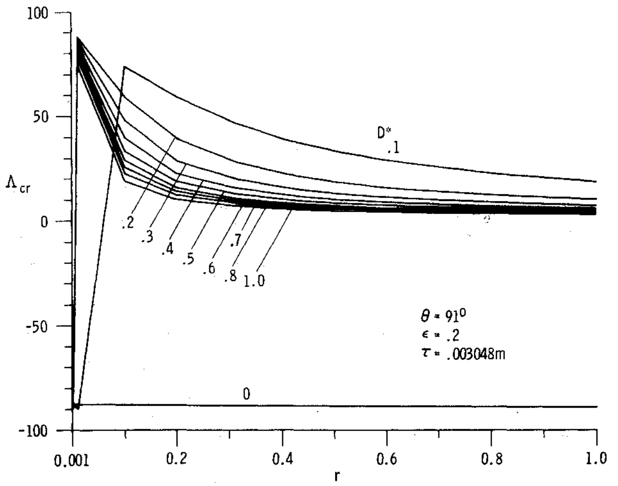

Fig. 14 Divergence-free wing sweep angle vs r ($\theta = 91$ deg).

divergence-free forward-swept angle of 63 deg is possible. Figure 14 shows that a wing designed with a composite material whose lamina properties, D^* or r, approach zero and are oriented at 91 deg, can be swept forward at any angle (up to 90 deg) without encountering real divergence speeds. Weisshaar's example[2] ($D^* = 0.319$, $r = 0.314$, $\epsilon = 0.36$) predicts a maximum divergence-free forward-sweep angle of 49 deg ($\theta = 102$ deg). Figures 17 and 18 compare the results of Ref. 2 and the present analysis ($\theta = 102$ deg). A comparison of Figs. 10 and 11 and Figs. 14-18 shows that while orientations in the 91-102 deg range (forward-swept fibers) are optimum for $r \to 1$, optimum orientations for $r \to 1$ might be in the 2-15 deg range (backward-swept fibers). This, therefore, is a generalization of a conclusion reached by Ref. 2 that only with forward-swept fibers ($\theta > 90$ deg) is a divergence-free forward-swept wing possible.

In terms of real materials, Table 1 (and similar data for other materials) could be used with the figures presented in this paper to determine the optimum configuration for a particular wing. For example, using Table 1 and Fig. 14, E-glass/epoxy would be the best candidate for an 80-deg forward-swept wing at $\theta = 91$ deg. Notice that in this analysis materials are characterized by their r, D^*, or ϵ values. Hence, a variation of the matrix for a particular material (e.g., graphite/epoxy), subject to manufacturing constraints, could produce the different "materials" required at various orientations (r is determined by the stiffness of the matrix relative to the fiber). Therefore, if these predictions are correct, this analysis has modified some of the conclusions of Ref. 2. Furthermore, an optimum aeroelastically tailored forward-swept wing is thus predicted to use different composite materials with various fiber orientations (this has an added advantage of improving the wing's strength).

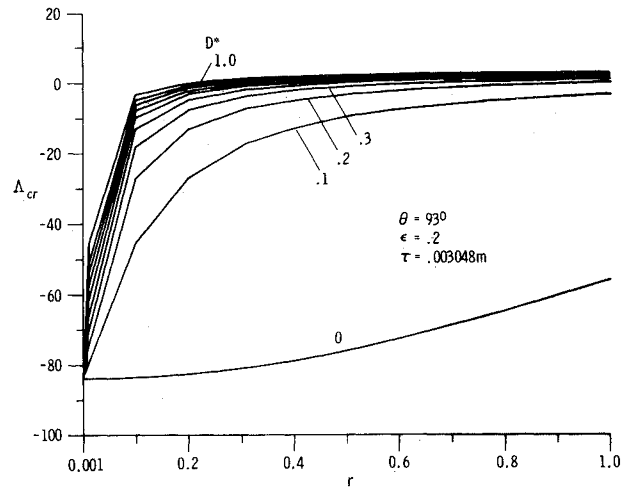

Fig. 15 Divergence-free wing sweep angle vs r ($\theta = 93$ deg).

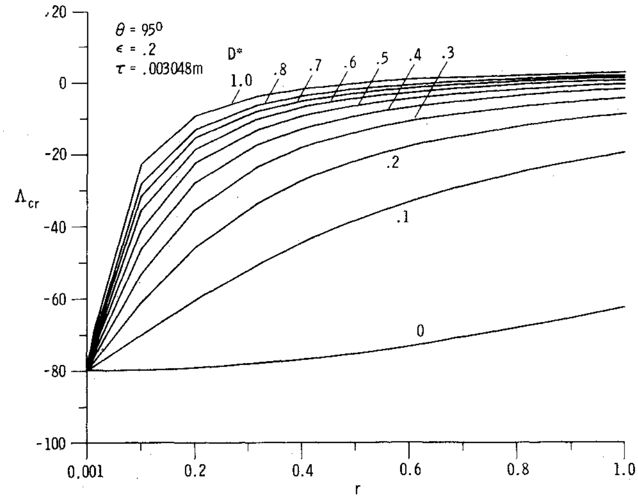

Fig. 16 Divergence-free wing sweep angle vs r ($\theta = 95$ deg).

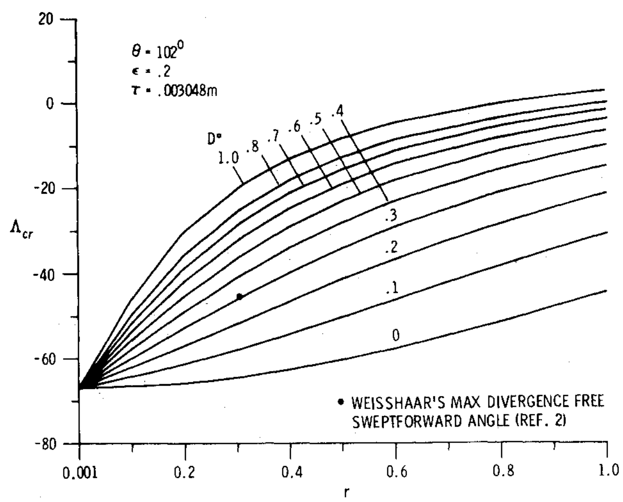

Fig. 17 Divergence-free wing sweep angle vs r ($\theta = 102$ deg).

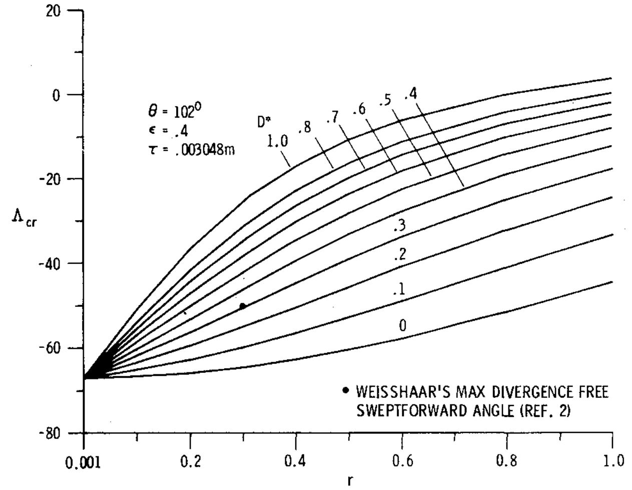

Fig. 18 Divergence-free wing sweep angle vs r ($\theta = 102$ deg, $\epsilon = 0.4$).

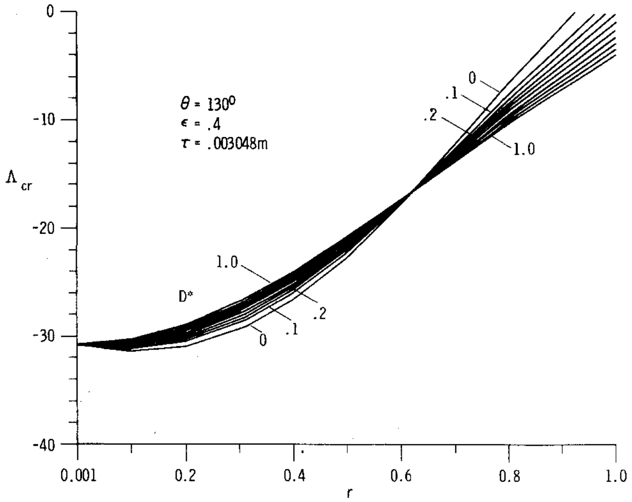

Fig. 19 Divergence-free wing sweep angle vs r ($\theta = 130$ deg, $\epsilon = 0.4$).

Concluding Remarks

A presentation of aeroelastic tailoring as a multivariable optimization problem has been attempted in this paper. A transformation of the aeroelastic equations of motion for swept wings reveals that the critical aeroelastic characteristics are expressible in terms of three bounded, generic stiffness variables and the fiber orientation angle. By varying these variables within their limits, a complete continuum of the critical divergence characteristics of forward-swept wings is presented for all composite materials. The results obtained appear to generalize various conclusions by previous investigators, which were based on a few example composite materials. Important new conclusions predicted by this investigation include: 1) divergence can be eliminated for a) any forward-swept angle, b) a forward-swept wing whose fiber orientation angles are swept back relative to the spanwise reference axis; and 2) an optimum aeroelastically tailored forward-swept wing is one that uses different composite materials oriented at various angles (a configuration that also enhances the wing's strength).

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